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Power distribution in developing countries — Planning for effectiveness and equity[☆]

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ABSTRACT

Power grid expansion planning is a challenging problem that typically considers the facility location, sizing, and transmission line upgrade aspects, with the objective of minimizing the upgrade and operational costs. The consideration of fairness and equity between the populations being served by the power grid has not been addressed previously in the literature. These issues are of special interest regarding the power grid infrastructure in developing countries, where certain populations might be "last in line" to be connected to the grid. In this paper, we develop a power grid expansion optimization model that considers both effectiveness and equity, given a budget constraint on upgrade expenditure. Effectiveness is measured by the deprivation costs of all populations served by the power grid, while equity is measured by their Gini mean absolute difference. Node upgrade rules are applied, and the upgrade plan is provided over a given planning horizon.

We optimally solved our model for small instances and performed sensitivity analysis on its parameters. We then developed an LNS (Large Neighborhood Search) heuristic for solving large instances and using publicly available data. Additional instances are generated based on the Myanmar power grid. Our analysis shows that the LNS can provide good solutions relative to a greedy approach.

The approach taken in this paper can be applied to a wide range of infrastructure planning problems in which both effectiveness and equity should be considered.

1. Introduction

Electricity is a daily necessity; in developed countries, it is supplied to the whole population. However, the supply of electricity in developing countries, where most of the population lives in rural areas, is often a privilege of the few. Technological developments of the last decade and the decreasing costs of technology allow us to provide these areas with solutions ranging from small personal renewable energy sources, energy storage, and micro-grids supplying whole villages to main power grid connectivity.

These different infrastructure solutions come with varying prices and benefits. For example, photovoltaic cells can provide electricity during the daytime. Coupled with investments in energy storage, the supply can be extended to the night. Full grid connectivity is much more expensive but can provide continuous supply.

The classical problem of power grid expansion planning has been extensively researched, see, for example, Sarid and Tzur [1] and Ghaddar and Jabr [2]. However, these papers did not include any fairness considerations. In addition, the classical problem assumes that all consumers are initially connected to the grid, and the goal is to then minimize the total infrastructure and/or generation costs, for example, Hemmati et al. [3], and Märkle-Huß et al. [4]. Another common approach is to maximize the grid's robustness, subject to budget constraints, for example, as in [5]. Unfortunately, in developing countries, not all consumers are connected or will be connected in the near future. Li [6] studies the interaction among three stakeholders: smart distribution networks, microgrids, and customers, whose coordination leads to a complex energy generation, storage, transaction, and consumption problem. They propose two coordination schemes and offer a mathematical model and solution method for each of them.

We find many similarities between the problem of planning a power grid expansion in developing countries and the field of humanitarian logistics, mainly the use of an objective that measures human suffering and equity between populations. Equity considerations are introduced via the objective function to balance two planning needs: effectiveness and equity. Such considerations serve as a guide toward solutions that also consider rural and non-central settlements. Specifically, areas that

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are not densely populated, and hence advancing them is less effective in terms of influence on the entire grid and the number of civilians affected by the upgrade.

In this paper, we consider the resource allocation problem for power grid expansion in developing countries. The goal is to achieve effectiveness with the expansion plan, i.e., to provide electricity as reliably as possible and to a population that is as large as possible. To measure effectiveness, we use the notion of deprivation costs — the equivalent monetary costs of not providing electricity to a specific population. The definition of deprivation costs is further explained in the literature review, Section 2, and also explored in Section 3. In addition to effectiveness, we also consider equity in power grid expansion planning. We measure equity by using the Gini mean absolute difference, which reflects the fairness of a policy. The Gini mean absolute difference and its relation to the Gini index are further explained in Section 3. The upgrade plan includes different stages over a duration of 10–30 years and the geographic layout of existing grid consumers.

An existing grid consumer cluster (i.e., a city, village, or other forms of settlement) is represented by a node in our problem. A node's current and future states are referred to as tiers and define the type of electricity supplied to the node. In our notation, a tier can represent a state such as full grid connectivity, a micro-grid solution, or even a no-power state. We need a node and its state over time to compute its deprivation costs. These concepts are defined carefully in Section 3.

Based on the above definitions, we formulate an optimization problem that minimizes an objective that considers effectiveness and fairness. The problem's solution yields a complete upgrade plan that specifies to the planner what node should be upgraded each year and to which tier. The goal is to achieve the minimum deprivation cost combined with a weighted Gini mean absolute difference for equity considerations. The problem adheres to constraints such as budget, node precedence, and additional upgrading rules.

We demonstrate our method on a small instance (of 10-nodes), which is solved to optimality, and develop an LNS meta-heuristic that is applied on larger instances (100 and 200-nodes). The larger instances are based on real-world data obtained from the Myanmar power grid, i.e., datasets published in [7] (2014–2015), and supporting information published by the World [8].

2. Literature review

As mentioned in the previous section, the problem of power grid expansion planning has been extensively studied from some perspectives (e.g., minimum cost, and robustness). However, the literature on grid infrastructure for developing countries is not as abundant. This is particularly the case in situations where both effectiveness and fairness play an important role. Karsu and Morton [9] reviewed applications of various OR problems in which measures of equity and balance play an important role, for example, in vehicle routing, scheduling, transportation, and supply chain design. According to Guo et al. [10], the enormous growth of China's solar sector is due to the strategy of combining it with poverty reduction, especially in rural areas.

In the field of humanitarian logistics, extensive developments have occurred in recent years. Holguín-Veras et al. [11] discussed post-disaster planning and introduced the concept of deprivation costs: the incurred costs of being deprived of a basic necessity, such as food, water, or power supply, after an extreme disaster event. Their model describes the hours, days, and weeks following a humanitarian disaster and the actions needed to provide relief. Eisenhandler and Tzur [12] used an objective function that includes a multiplicative combination of 1 minus the Gini index and an effectiveness measure in the context of food rescue via food bank donations. Gutjahr and Fischer [13] extended the concept by considering another element of equity: the Gini mean absolute difference (not to be confused with the Gini index or Gini coefficient). They treated the post-disaster supply problem by defining the supply frequency as the decision variable. Cantillo et al. [14] suggested an econometric approach (using a discrete choice experiment) for estimating the deprivation cost functions.

Barbati and Piccolo [15] and Noham and Tzur [16] discussed various additional equity objective functions or constraints in the context of facility location. Dönmez et al. [17] provided a comprehensive review of the research on facility location problems in a humanitarian context, including the type of facilities involved, the decisions that need to be made, the criteria to optimize, and the solution method adopted.

Focusing on equity in infrastructure planning, Mostajabdaveh et al. [18] considered the problem of positioning shelters for emergencies. Their main consideration was the mean distance between opened shelter locations and individuals with the objective of minimizing inequity under uncertainty. In [19], the problem of equity in power grid design was discussed from the firms' perspective, i.e., when the market is deregulated to several utility companies or distributors, such that each firm has commercial viability. This perspective is very different from the one we consider in this paper, which discusses equity from the consumer's perspective.

Flores et al. [20] analyze a 2021 Texas power crisis in which a large portion of the population suffered a power shortage. Disparities during this crisis are estimated using utility data, customer surveys, and spatial analysis to compare the severity of the outage. Their analysis indicates that an uneven burden of the power shortage exists. Similarly, Liévanos and Horne [21] deal with the inequality of power grids from a resilience perspective, i.e., the duration of electricity outages in the US compared by various sociodemographic variables. They also indicate inequalities that seem to derive from prioritizing specific assets (e.g., hospitals).

To the best of our knowledge, no study so far has examined equity and fairness in power grid infrastructure planning. Thus, the current paper brings a completely new perspective to this topic, using definitions and performance measures that have already been developed for pre-disaster and infrastructure planning in humanitarian logistics. In particular, we adopt the approach that examines equity in the context of the Gini mean absolute difference of deprivation costs. We suggest a model for the optimization problem and conduct several numerical experiments with that model.

The next section provides a detailed description of the problem we consider. Section 4 details the LNS (Large Neighborhood Search) solution method we use for the larger instances. In Section 5, we present numerical experiments on randomized and real-world based instances, and then, in Section 6, we provide conclusions with possibilities for future work.

3. Problem description and formulation

In the problem we consider, there exists a partial power grid, i.e., there are urban areas connected to the main power grid and rural areas that are not connected and are either completely without electricity or with limited electricity, which fulfills only a portion of the demand. For example, a household may have electricity provided by a self-sustained photo-voltaic cell, and a village might also be connected to a micro-grid.

We represent the demand in the grid using nodes. A node may represent a village, a city, or even an entire region (e.g., a cluster of a few similar villages). Given the existing grid, we aim to find an upgrade solution for the nodes, indicating which nodes should be upgraded and when. The planning horizon itself may consist of a duration of 10–30 years. The upgrade plan is subject to constraints such as monetary constraints (a limit on the annual and on overall spending), upgrade rules (which nodes can be technically upgraded), and precedence constraints.

The nodes transition between states/tiers, where each node incurs a deprivation cost according to the tier it belongs to. This represents the cost for the population living at that node not being able to connect to the power grid or having an unreliable power supply. We aim to minimize the weighted average of the deprivation cost (over all nodes and the entire planning horizon), along with a fairness (equity) element.

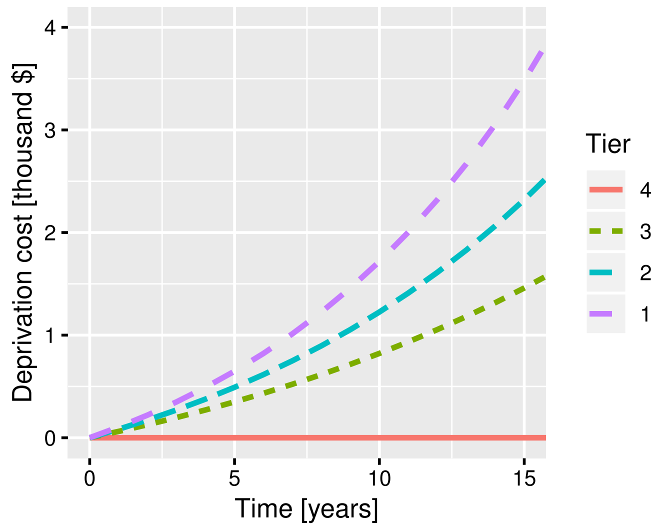

Fig. 1. Deprivation cost of different tiers as a function of time. The y-axis reflects the cost (illustrative), and the x-axis the years.

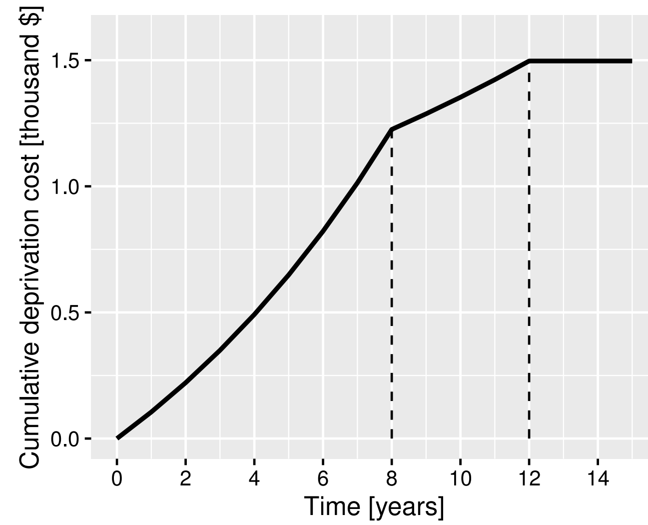

Fig. 2. The total cumulative deprivation cost over a 15-year horizon at a node that shifts through different tiers.

3.1. Measuring deprivation cost and equity in power grid development

We assume that the deprivation cost is a non-decreasing (and convex) function of time, i.e., as years go by and a certain node remains in the same tier, its yearly deprivation cost increases. This assumption follows Holguín-Veras et al. [11] which specifically use an exponential deprivation cost and also exhibit a piecewise linear discretization of that function. The justification of an exponential function is that the socio-economic disadvantages of a population deprived of electricity supply grow deeper with each year of exclusion. The deprivation is measured in currency (e.g., $) and can be assessed, for example, using the techniques presented in [14].

In our case, we assume that there are four possible tiers for electric supply, in the following order:

- Tier 1: No electricity supply.
- Tier 2: Partial-low electric supply — provided by local solutions (e.g., photo-voltaic cells).
- Tier 3: Partial-high electric supply — provided by regional solutions (i.e., a micro-grid).
- Tier 4: Full electric supply — full grid connectivity (connection to the main grid).

Each tier has its own deprivation cost, i.e., a population with no electric supply has the highest deprivation cost (because they are completely deprived of electricity). Consumers with a partial electric supply have a lower deprivation cost: they might have electricity for part of a day (e.g., in the daytime) but at a low capacity (low Wattage) and perhaps less reliable. Consumers with a full electric supply have no deprivation cost (cost is zero) since they have a reliable electricity supply at a high capacity. In addition, annual deprivation costs increase over time.

Fig. 1 illustrates possible deprivation costs at the various tiers. The figure shows Tier 4 without any cost (full grid connectivity) and Tier 1 with the highest cost. Fig. 2 illustrates the cumulative deprivation cost at a node that transitions from Tier 1 to Tier 2 in year 8 and again from Tier 2 to Tier 4 in the 12th year. From the 12th year to the end of the planning horizon, the node is at the highest possible tier and, therefore, does not accumulate additional deprivation costs.

3.2. Definitions and assumptions

First, we define node precedence.

Definition 1 (*Node Precedence*)**.** Node j has a tier t precedence to node i, if a prerequisite to upgrade node i to tier t is that node j is upgraded (or has been upgraded) to at least tier t. We denote this relation as:

$$j \overset{t}{\rightarrow} i \tag{1}$$

If any of the nodes $\{j_1, \ldots, j_n\}$ has a tier t precedence to node i, we denote this as:

$$j_1 \vee \ldots \vee j_n \overset{t}{\rightarrow} i \tag{2}$$

This definition would apply in situations when a power line established from the main grid to a specific target node i, has to go through another node (one of $\{j_1, \ldots, j_n\}$) which is on some path to the target node i. In other words, at least one of the nodes $j_1, \ldots, j_n$ should be upgraded to tier t (no matter which one) before or at the same time node i is updated to tier t. This definition is applicable mostly for upgrades to the highest tier, though in some cases, it may also be relevant for other tiers.

Assumption 1 (*Node Upgrade Precedence*)**.** We assume that a hierarchy of node upgrade precedence constraints can be expressed as an input to the problem. Furthermore, we assume that when $j_1 \vee \ldots \vee j_n \overset{t}{\rightarrow} i$, the cost for upgrading i does not depend on which of the nodes $j_1, \ldots, j_n$ facilitated the upgrade of i.

Assumption 2 (*Additive and Memoryless Deprivation Cost*)**.** When a node is upgraded to a new tier t, its deprivation cost until the next upgrade (if it occurs) is independent of the history of its previous tiers and the respective duration it has spent in each previous tier.

We define the deprivation cost for node i being in tier t for a duration of d years by $g_{it}(d)$. The cumulative costs according to Assumption 2 and the node upgrade illustrated in Fig. 2 are provided below. The node in the figure is upgraded from Tier 1 to Tier 3 at year 8 and then upgraded to Tier 4 at year 12. The total deprivation cost of this node (denoted as node i) up to year y is given by:

$$\text{Tot. Cumulated Deprivation}_i(y) = \begin{cases} g_{i1}(y) & y \leq 8 \\ g_{i1}(8) + g_{i3}(y-8) & 8 < y \leq 12 \\ g_{i1}(8) + g_{i3}(4) & \text{Otherwise} \end{cases}$$

The cost up to year y for $y \leq 8$ is given by the deprivation cost of Tier 1 for the duration of y years, i.e., $g_{i1}(y)$. The additional cost in the range $8 < y \leq 12$ is given by $g_{i3}(y-8)$, and the additional cost during

$y > 12$ is 0, since the highest tier has been attained at $y = 12$ and it does not incur any additional cost.

Assumption 3 (*Identical Per Node Deprivation Costs*)**.** All individuals/consumers represented by a certain node are assumed to be identical, i.e., for a given tier t, all consumers in node i have the same deprivation cost function.

In other words, Assumption 3 means that the functions depicted in Fig. 1 are identical for all consumers represented by the same node.

3.3. Formulation as an optimization problem – The power grid equity expansion problem

Let index t represent the possible tiers $t \in \{1, \ldots, T\} = \mathcal{T}$. Let y represent the years in the planning horizon $y \in \{0, \ldots, Y\} = \mathcal{Y}$, and i be a node at which there is electricity demand, $i \in \{1, \ldots, N\} = \mathcal{N}$.

Node i represents the population of a city, town, village, or rural location, where all individuals/consumers in the node are identical in the sense that they require the same amount of electricity and have the same deprivation cost. This simplifying assumption ensures that deprivation cost can be expressed using a weighted average of nodes. In fact, we require that the deprivation cost be proportional to the population size, and we do not need individuals to be identical.

The parameters of the problem are:

- π_i, the weight of node i, i.e., the total population at node i divided by the total population in the country. Note that by definition, $\sum_i \pi_i = 1$.
- B_y, the annual budget which is given to the planner at the beginning of year y.
- $c_{it_1t_2y}$, the cost of upgrading node i from tier t_1 to tier t_2 at the beginning of year y.

Recall that we used the function $g_{it}(d)$ to represent the deprivation cost for node i being in tier t for a duration d years.

The decision variables are defined $\forall i, t, y$:

$$X_{ity} = \begin{cases} 1 & \text{if node } i \text{ is at tier } t \text{ during year } y \\ 0 & \text{Otherwise} \end{cases}$$

To express upgrade decisions at the beginning of each year, we use $\forall i, y, t_1 < t_2$:

$$U_{iy}(t_1, t_2) = \begin{cases} 1 & \text{if node } i \text{ is upgraded from tier } t_1 \text{ to } t_2 \\ & \text{at the beginning of year } y \\ 0 & \text{Otherwise} \end{cases}$$

We use an auxiliary decision variable D_{it} to denote the duration of node i in tier t, i.e., $D_{it} = \sum_{y=0}^{Y} X_{ity}$. Then, the term $\sum_t g_{it}(D_{it})$ represents the total accumulated deprivation cost of node i.

Let $\vec{X}$ denote a feasible solution in terms of the X_{ity} decision variables, i.e., $\vec{X}$ represents the entire upgrade plan for all nodes, tiers, and years. To consider equity in the grid upgrade plan, we need some measure of fairness or dispersion with respect to the deprivation cost incurred by each node while also considering the weights of the nodes. We shall use the Gini mean absolute difference $\Delta_{\text{Gini}}(\vec{X})$.

$$\Delta_{\text{Gini}}(\vec{X}) = \sum_{i,j \in \mathcal{N}} \pi_i \pi_j \left| \sum_t g_{it}(D_{it}) - \sum_t g_{jt}(D_{jt}) \right| \tag{3}$$

The Gini mean absolute difference quantifies equity using the sum of the difference over all pairs of nodes of a certain measure (such as wealth or deprivation cost in our case) while weighing each difference. The multiplication by $\pi_i \pi_j$, which appears in Eq. (3), weighs the difference between i and j. Intuitively, larger differences express inequity, but thanks to the multiplication by $\pi_i \pi_j$, large differences between negligible nodes (with small populations) are less influential than differences between large nodes. As the value of Δ_{Gini} decreases, the solution becomes more equitable (see also [13], who use the Gini mean absolute difference in a similar manner).

Additional benefits to using the Gini mean absolute difference are that it is based on the deprivation cost and tiers of all of the nodes (not just extreme values) and that it is closely related to the Gini index (the numerator of the Gini mean absolute difference), which is widely used in economic problems to measure inequity.

The Gini index is always between 0 (when all nodes have identical values, i.e., complete equity) and 1 (when only one node has a positive value, i.e., complete inequity) and is defined by:

$$G(\vec{X}) = \frac{\Delta_{\text{Gini}}(\vec{X})}{2 \sum_{it} \pi_i g_{it}(D_{it})}$$

However, the Gini index is not linear, and this complicates the efforts to define an objective function that is computationally tractable. Hence, we adopt the approach of Gutjahr and Fischer [13], who use the Gini mean absolute difference, i.e. the expression in Eq. (3). The Gini mean absolute difference is included in the objective function using the multiplier λ, along with the expression which reflects the average deprivation cost. The Gini mean absolute difference is relatively easy to linearize within the objective function.

We now present the formulation in full, followed by a detailed explanation of the constraints.

The goal is:

$$\min \sum_{t,i} \left[\pi_i \cdot g_{it} \left(\sum_{y=0}^{Y} X_{ity} \right) \right] + \lambda \Delta_{\text{Gini}} \left(\vec{X} \right) \tag{4}$$

subject to the following constraints:

$$\sum_{t=1}^{T} X_{ity} = 1 \qquad \forall i \in \mathcal{N}, \forall y \in \mathcal{Y} \tag{5}$$

$$\sum_{\tau=t}^{T} X_{i\tau y} \geq X_{ity-1} \qquad \forall y \geq 1, \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{6}$$

$$X_{ity} \leq \sum_{l=1}^{L} \sum_{\tau=t}^{T} X_{j_l \tau y} \forall t, \qquad \forall y, \text{ and } j_1 \vee \ldots \vee j_n \xrightarrow{t} i \tag{7}$$

$$\sum_{t=1}^{T} t X_{ity} - \sum_{t=1}^{T} t X_{ity-1} = \sum_{1 \leq t_1 < t_2 \leq T} (t_2 - t_1) U_{iy}(t_1, t_2) \qquad \forall i \in \mathcal{N}, \forall y : 1 \leq y \in \mathcal{Y} \tag{8}$$

$$\sum_{t_2 : t_1 < t_2 \leq T} U_{iy}(t_1, t_2) \leq X_{it_1 y-1} \qquad \forall i \in \mathcal{N}, \forall t_1 \in \mathcal{T}, \forall y : 1 \leq y \in \mathcal{Y} \tag{9}$$

$$\sum_{y' \leq y} \sum_i \sum_{t \leq t'} \left[c_{it_1t_2y'} U_{iy'}(t_1, t_2) \right] \leq \sum_{y' \leq y} B_{y'} \qquad \forall y \in \mathcal{Y} \tag{10}$$

$$X_{it_{i,\text{init}}0} = 1 \qquad \forall i \in \mathcal{N} \tag{11}$$

$$U_{iy}(s, t') = 0 \qquad \forall s < t' < t_{i,\text{init}}, \forall y \in \mathcal{Y} \tag{12}$$

The objective function in (4) is comprised of two elements. The first element relates to the solution's effectiveness and is measured in average deprivation cost (i.e., $\sum_{t,i} \left[\pi_i \cdot g_{it} \left(\sum_{y=0}^{Y} X_{ity} \right) \right]$). The second element is the aforementioned Gini mean absolute difference, which measures the inequity. The Gini mean absolute difference is multiplied by λ in the objective function. The λ parameter can be increased or decreased to reflect the importance that the planner/decision-maker wants to give to equity.

The constraints (5) ensure that every node is assigned to exactly one tier at each time in the planning horizon. The constraints (6) make sure that it is only possible to upgrade a node from a low tier to a higher tier.

As Definition 1 states, we would sometimes like to require a node upgrade to be permissible only when certain other nodes reach a specific tier. This is what the constraints (7) take care of. In the formulation of (7), any one of $j_1, \ldots, j_n$ is enough to allow the upgrade of i to tier t.

Constraints (8) and (9) connect the X_{ity} variables to their respective upgrade decision variables $U_{iy}(t_1, t_2)$. They work in the following manner: when no change in tiers is performed (when moving from $y-1$ to y), the respective tX_{ity} and tX_{ity-1} at the LHS of (8) cancel each other, such that all the upgrade decision variables in the RHS must equal zero. When an upgrade from tier t_1 to t_2 is performed for node i in year y, we will have a value of $t_2 - t_1$ on the LHS, and one of the variables in the RHS which is multiplied by a value of $(t_2 - t_1)$, indicating an upgrade of $t_2 - t_1$ tiers. However, there might be more than one such decision variable. The constraints (9) make sure that at most only one of the $U_{iy}(t_1, \cdot)$ variables will have a positive value (because their sum is bounded by an indicator which is at most one). Hence, the two constraints combined make sure that only the correct $U_{iy}(\cdot, \cdot)$ variable will be positive, and the rest will be zero.

Constraints (10) set the budget: tier upgrades at a specific year y cannot exceed the total budget B_y for that year, along with the unused budget in previous years. Note that money unspent in one year can be spent in a future year. Constraints (11) make sure that each node is initialized in its starting conditions, i.e., at its actual tier at the beginning of the planning horizon: $t_{i,\text{init}}$ at year 0. Constraints (12) make sure that a node cannot be upgraded between tiers under its initial tier.

In Appendix A, we explain how to modify the formulation and turn it into a mixed integer problem (MIP), by linearization of the deprivation costs.

We emphasize that the λ parameter in the objective function reflects a decision maker's choice about balancing effectiveness versus equity. It is interesting to study the effect that λ has on the optimal solution and on each of the two terms in the objective. We highlight a few monotonicity properties regarding the Gini mean absolute difference, the deprivation costs, and the Gini index as a function of λ.

Lemma 1. *As λ increases, the value of the Gini mean absolute difference in the optimal solution is non-increasing, the value of the average deprivation cost in the optimal solution is non-decreasing, and therefore the Gini index is non-increasing.*

Proof. Assume λ_1 and λ_2, $0 < \lambda_1 < \lambda_2$. Let $\vec{X}_k$ represent the optimal solution attained by setting λ_k $(k = 1, 2)$. We use the notation g_k, Δ_k, G_k to represent the average deprivation cost, the Gini mean absolute difference, and the Gini index, respectively, attained at the optimal solution when using λ_k, $k = 1, 2$.

First note that due to optimality of $\vec{X}_1, \vec{X}_2$, the following holds:

$$g_1 + \lambda_1 \Delta_1 \leq g_2 + \lambda_1 \Delta_2 \tag{13}$$

$$g_2 + \lambda_2 \Delta_2 \leq g_1 + \lambda_2 \Delta_1 \tag{14}$$

Considering $g_1 > g_2$ or $g_1 < g_2$ we obtain (15) and (16) respectively:

$$0 < g_1 - g_2 \leq \lambda_1(\Delta_2 - \Delta_1) \Rightarrow \Delta_2 > \Delta_1 \tag{15}$$

$$0 < g_2 - g_1 \leq \lambda_2(\Delta_1 - \Delta_2) \Rightarrow \Delta_1 > \Delta_2 \tag{16}$$

By combining (15) and (16) we obtain $g_1 < g_2 \iff \Delta_2 < \Delta_1$, and also:

$$\lambda_2(\Delta_2 - \Delta_1) \leq g_1 - g_2 \leq \lambda_1(\Delta_2 - \Delta_1) \tag{17}$$

Since we assumed $0 < \lambda_1 < \lambda_2$, Eq. (17) indicates that $\Delta_2 < \Delta_1$, and therefore we obtain $g_2 > g_1$. Since both g_k, Δ_k are non-negative we obtain:

$$G_1 = \frac{\Delta_1}{g_1} \geq \frac{\Delta_2}{g_2} = G_2 \tag{18}$$

Hence, when λ increases, the Gini mean absolute difference is non-increasing, the deprivation cost is non-decreasing, and the Gini index is non-increasing. □

4. An LNS heuristic algorithm for the power grid equity expansion problem

The problem we address can be shown to be NP-Hard by reducing a special case of it to the knapsack problem. The special case has n nodes and only two periods and two tiers. Then, the decision is which nodes to upgrade to the higher tier in the first period, where node-specific upgrade costs exist, and similarly, node-specific benefits (cost savings in the second period) from upgrading it. Then, deciding which nodes to upgrade under the period's budget constraint reduces to the knapsack problem. Thus, to solve large-scale instances, we employ an LNS meta-heuristic approach. Following the Algorithm suggested in [22], we initialize with a feasible greedy solution and then search for improved solutions using "destroy and rebuild" operations. Note that the terms "destroy" and "rebuild" relate to algorithmic steps regarding the solution's feasibility rather than the grid's infrastructure. I.e., destroying (rebuilding) a solution means upgrading (downgrading) different elements of the solution until it is no longer feasible (feasible) in terms of budget. These are standard terms in the LNS literature.

A solution is defined as a two-dimensional array: one dimension is the size of the planning horizon, and the other is the number of nodes. The array values describe the grid states at each year and node (i.e., the node's tier at each year). That is, the value in position year y, node i in the array is the equivalent of $\sum_t tX_{ity}$ (which were defined in the formulation of the problem in the previous section). Even though it is possible to further reduce this representation to include only the occasions in which a tier was changed (instead of the entire grid state), we chose to use a full representation of the grid for convenience.

An upgrade decision is defined as increasing the tier of a specific node i at a specific year y (and updating the grid in the following years accordingly). The upgrade mechanism is subject to the same constraints described in (5)–(9); however, it is not subject to the budget constraints (10) which will be considered in future steps of the algorithm.

A downgrade decision is defined as lowering the tier of a specific node i at a specific year y (and updating the grid in future years accordingly). A downgrade cannot be to a lower tier than the previous tier the node was in (prior to year y).

4.1. Initialization of the greedy solution

For the initialization of the greedy solution:

1. We map all the possible and feasible (in terms of budget) upgrade decisions from the current grid state (i.e., create a list of upgrade steps) and calculate the objective function value associated with each upgrade step.
2. We iteratively choose one upgrade step at a time from the list. The upgrade step that is selected is the step that has the lowest objective value of all possible upgrades on the list.
3. After each selection, we omit the items from the list that are no longer relevant (i.e., because of an upgrade that was already applied or due to budget constraints).
4. The process continues until the list is empty.

The destroy and rebuild operations are then employed interchangeably.

4.2. The destroy function (violate feasibility)

In the following description, the term "destruction" relates to the action of upgrading until the solution is no longer feasible because the budget constraints are violated.

1. Similarly to the step described in the initialization of the greedy solution, we map all the possible upgrade decisions from the current grid state and calculate the expected objective function value associated with each upgrade step.

2. We associate a weight to each upgrade step, such that the weight decreases as the objective value increases.
3. We randomly choose an integer, destruction degree, ranging between one and a destruction degree upper bound (a parameter of the algorithm).
4. We randomly choose upgrade steps according to their weights until the budget constraints are violated, and then choose additional upgrades according to the destruction degree chosen in the previous step.

4.3. The rebuild function (regain feasibility)

In the following description, the term "rebuild" relates to the fact that by applying enough steps from the "rebuild steps" list, some upgrades are omitted, less budget is used, and no budget constraints are violated. Feasibility is regained during this step.

1. A list of possible "rebuild steps" is generated, i.e., downgrade steps that influence the excess use of budget in the violating solution.
2. Downgrade steps are drawn until feasibility is restored (i.e., budget constraints are not violated throughout the planning horizon).

5. Numerical study

The purpose of this numerical study is two-fold: in the first part of this numerical study, we use a small instance that is solved to optimality to examine the influence of the λ and budget parameters on the resulting upgrade plan. We also compare the optimal solutions to the LNS meta-heuristic's solutions on the same instances. We provide this analysis using instances with varying deprivation costs and available budgets. In the second part of this numerical study, we use larger instances (100 and 200-nodes) which are solved only with the LNS meta-heuristic, and examine this solution's objective value compared to a greedy solution's objective value (to see the contribution and performance of the LNS).

An analysis of the 10-node example is provided in Section 5.1, followed by a case study which is based on the Myanmar power grid, provided in Section 5.2 with 100 and 200-node instances.

5.1. The 10-node example

This small example was generated to make it easy to visualize, analyze, and perform sensitivity analysis. The grid associated with the example is illustrated in Fig. 3, where the arrows' directions represent node precedence. For example:

- Node 10 can be upgraded to Tier 2, but to upgrade it to Tier 3 or 4, Node 6 must also be upgraded to Tier 3 or 4, respectively.
- Node 7 can be upgraded to Tier 3 if and only if Node 8 is also upgraded to Tier 3. That is, they must be upgraded to Tier 3 together. The inspiration for such two-way precedence is a micro-grid that is shared by several nodes.
- Node 5 can be upgraded to Tier 4 if either Node 3 or Node 4 is at Tier 4. That is, one or both of them should already be at Tier 4 or upgraded to Tier 4 in the same year Node 5 is upgraded to it.

The size of each node in this grid represents the relative proportion of the node's population, and the node's color (or darkness) represents the tier to which it belongs initially.

We can see that two nodes (1, 2) containing 50% of the population are initialized to Tier 4, while three other nodes (3, 4, and 5) are initialized to Tier 3 and contain 30% of the population. Another three nodes (6, 7, and 8) are initialized to Tier 2 and contain 15% of the population. The remaining nodes (9 and 10) are initialized to Tier 1 and contain only 5% of the population. In the figure, the x-y coordinates of the nodes have no actual meaning.

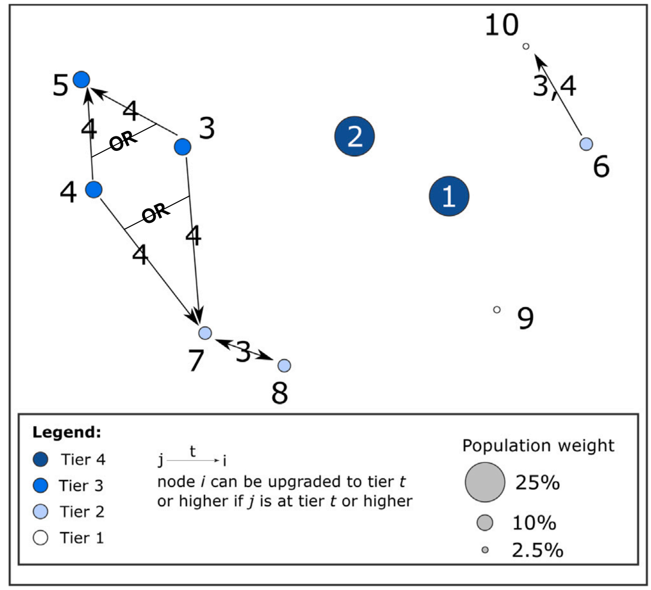

Fig. 3. The network of our sample instance with ten nodes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Deprivation costs and upgrade costs are uniform between nodes and years, that is, they only depend on the node's tier. These costs are given in Tables 1 and 2 respectively. Throughout the numerical study, we do not include the cost incurred at the initial state (year 0) so that given an infinite budget, the deprivation costs are 0.

At each tier level, we assume that the deprivation cost function takes the form $e^{\theta \times y} - 1$ as a function of y. The value of θ depends, of course, on the tier. Note that this is a dimensionless quantity, which should be multiplied by a factor to measure the deprivation costs in monetary units. However, such a factor would not play a role since the objective function is the only element within the model, which includes the deprivation cost, and other costs (the upgrade costs) appear only in the constraints.

We consider five cases for the four-tier values of θ; see Table 1. The first is the base deprivation cost, and the other three variations are multiplications of the deprivation cost parameter by $\times 2$, $\times 1/2$, and $\times 1/4$. Another variation (i.e., the "tenfold" instance) contains $\theta \in \{0, 1, 10, 100\}$ according to the tier.

Regarding the upgrade costs in Table 2, note the following observations (costs in thousands of dollars):

- Upgrading from Tier 1 or Tier 2 to Tier 4 has the same cost ($55). In this case, an upgrade from Tier 2 does not reduce the cost of an upgrade to Tier 4.
- Upgrading from Tier 1 or Tier 2 to Tier 3 has the same cost ($15). A similar explanation as in the previous case also applies here.
- Upgrading from Tier 3 to Tier 4 is slightly less expensive than upgrading from Tier 1 or 2 to Tier 4 ($45 versus $55). This is because it is assumed that some of the infrastructure has already been laid down on the upgrade to Tier 3.

We used the linearized version of the power grid equity expansion optimization problem to find the optimal expansion plan for the above instance while using the following parameters:

- Planning horizon of 15 years.
- λ varies in $\{0, 0.25, 0.5, 1, 3, 30, 1000\}$. When $\lambda = 0$, no weight is given to equity, so only deprivation costs are minimized.

Table 1
The base variation of deprivation costs. The values in the table describe the θ parameter of an exponential deprivation cost, i.e., $e^{\theta \times y} - 1$.

Deprivation cost					
Multiplications of θ					Tenfold
Tier	Base	Double	Half	Quarter	Variation
1	0.1	0.2	0.05	0.025	100
2	0.08	0.16	0.04	0.02	10
3	0.06	0.12	0.03	0.015	1
4	0	0	0	0	0

Table 2
Upgrade costs between tiers for our sample instance with ten nodes.

Tier		Upgrade cost
From	To	[thousand $]
1	2	5
1	3	15
1	4	55
2	3	15
2	4	55
3	4	45

- Budget varies in $\{300, 500, 1000\}$ (this is the total budget over the planning horizon so that the annual budget is divided by 15).

5.1.1. Results for the 10-node example

To compare the resulting upgrade plans, we use the Gini index with respect to the incurred deprivation cost at the nodes. The Gini measure is commonly used to examine inequity between populations. Even though it does not directly appear in our objective function, it can represent both measures used in our objective: as noted earlier, it is the ratio between the second element (the Gini mean absolute difference) to the first element of our objective (average deprivation cost). Among other things, we will explore the rate of decrease of the Gini index when the multiplier λ on the Gini mean absolute difference increases.

The instances were solved to optimality on an i9-9900K @ 3.6 GHz, an 8-core server running IBM CPLEX 12.9 within the allotted time frame of three hours (and, in most cases, in only a few minutes). The results are given in Table 3: each row relates to a different combination of deprivation cost variation and budget. Each column (starting from the third column) indicates a different λ. In total, the solutions of 90 instances are shown in the table. The numbers within the table indicate the Gini index of the solution. The Gini index values in boldface indicate that a decrease in the Gini index is observed as compared to the previous λ value.

As expected, the Gini index values of the obtained solutions are non-increasing in the λ parameter. This numerically validates the statement of Lemma 1. In certain cases, increasing the λ parameter does not change the solution (and the Gini index does not change). However, increasing this parameter in other cases caused the Gini index to decrease. For example, in the quarter cost deprivation variation, budget 300, we see that the Gini index decreases for at least two λ values: once for $\lambda \in (1, 3]$ and the second time when $\lambda \in (3, 30]$. Changes at $\lambda = 1000$ were not detected in any of the instances (hence, this column is not included in Table 3).

To examine the behavior of the Gini index as a function of λ at an even higher resolution, we singled out a specific instance and explored the optimal solution with additional λ values. The Quarter Cost instance, with budget 300, and $\lambda \in \{0, 0.25, 0.5, 1, 2, 3, 5, 7.5, 8, 9, 10\}$ was chosen due to the changes observed in Table 3 for $\lambda = 1, 3, 30$ resulting in Gini indices of 0.703, 0.692, and 0.609 respectively. We found that the decrease to the minimal Gini index is actually already apparent for $\lambda = 8$ and occurs within the interval $\lambda \in (7.5, 8]$, as can be seen in Fig. 4. This subtle change is, in fact, derived by meaningful changes to the upgrade plan, as illustrated by Fig. 4. For $\lambda > 8$ the solution remains the same as for $\lambda = 8$. For $\lambda < 1$ the solution is the same as in $\lambda = 1$, the change occurs within the interval $\lambda \in (1, 2]$. For $\lambda = 2$, the solution is the same as in the $2 < \lambda \leq 7.5$ range. This justifies using the λ parameter in the objective function as an equity weight parameter that can be tuned.

Table 3
Gini index as a function of λ for the 10-node example.

Instance	Budget	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 3$	$\lambda = 30$
Base variation	300	**0.715**	0.715	0.715	**0.695**	0.695	0.695
Base variation	500	**0.737**	**0.733**	0.733	0.733	0.733	**0.732**
Base variation	1000	**0.766**	**0.765**	0.765	0.765	**0.761**	0.761
Double cost	300	**0.704**	0.704	**0.694**	0.694	0.694	0.694
Double cost	500	**0.742**	0.742	0.742	0.742	0.742	**0.731**
Double cost	1000	**0.771**	**0.765**	0.765	0.765	0.765	0.765
Half cost	300	**0.707**	0.707	0.707	0.707	0.707	**0.613**
Half cost	500	**0.733**	0.733	0.733	**0.727**	0.727	0.727
Half cost	1000	**0.762**	**0.758**	0.758	0.758	0.758	0.758
Quarter cost	300	**0.703**	0.703	0.703	0.703	**0.692**	**0.609**
Quarter cost	500	**0.731**	0.731	0.731	**0.729**	**0.726**	**0.678**
Quarter cost	1000	**0.760**	**0.756**	0.756	0.756	0.756	0.756
Tenfold cost	300	**0.693**	**0.69**	0.69	0.69	0.69	**0.657**
Tenfold cost	500	**0.705**	**0.698**	**0.686**	**0.684**	**0.683**	**0.642**
Tenfold cost	1000	**0.681**	**0.68**	0.68	0.68	0.68	**0.634**

We use this example to examine the properties and differences between the solutions of the two extreme cases, where $\lambda = 1$ versus $\lambda = 8$. Fig. 5 shows the node upgrade decisions for the two λ values where each box represents a different node. Nodes 1 and 2 started at the highest tier (4) and clearly remain at that tier, therefore excluded from the figure. The x-axis represents the years (0 is the initial year, and 14 is the maximum year), and the y-axis represents the tier level (1–4).

As noted in Fig. 4, when λ increases from $\lambda = 1$ to $\lambda = 8$, there is a change in the Gini index: a decrease of 13.4% from 0.703 to 0.609. This subtle change is, in fact, derived by meaningful changes to the upgrade plan: the actual difference between the two solutions is apparent in all nodes. For example, when $\lambda = 8$, Node 9 is upgraded much earlier (year 1) and to a higher tier (Tier 3) compared to $\lambda = 1$ (year 8, Tier 2). Since this node represents a small population that starts at a low tier, an early upgrade of it increases the overall equity. A similar effect can be observed in nodes 7–8 and 10. In contrast, nodes 4 through 6 are upgraded later when $\lambda = 8$. Since these nodes start from a higher level (Tier 3), postponing their upgrade increases the equity. Thus, the increase of λ brought more upgrades (sooner and higher) to the smaller nodes (with less population), which were at a lower tier.

With respect to the budget, in most cases, we observe that the Gini index increases as the budget increases. This is because as the budget increases, additional upgrade opportunities open up, and upgrades are applied earlier in the planning horizon and across the board. When nodes are upgraded sooner, specifically nodes with large populations, the equity decreases, compared to more budget-constrained cases where emphasis can be put on the smaller population nodes. However, it is hard to generalize the influence of the budget on the Gini index. It may increase or decrease as the budget increases. For example, in the tenfold instance (the last three lines in Table 3), as the budget increases from 300 to 500 and then to 1000, combined with $\lambda = 0.5$, the Gini index decreases.

With respect to the deprivation parameters, two factors characterize the instance: the parameters' value and the ratio between tiers (i.e., in the first four groups of instances of Table 1, the ratio is the same, and the absolute value changes, and in the last "tenfold" group of instances, the ratio is 10). We notice that as the value of the parameters increases (in the first four groups), the Gini index also increases in most cases. However, the last group of instances exhibits a lower Gini index overall. We believe this is due to the extreme cost of maintaining a node at a lower tier, which encourages its early upgrade, which in turn increases equity.

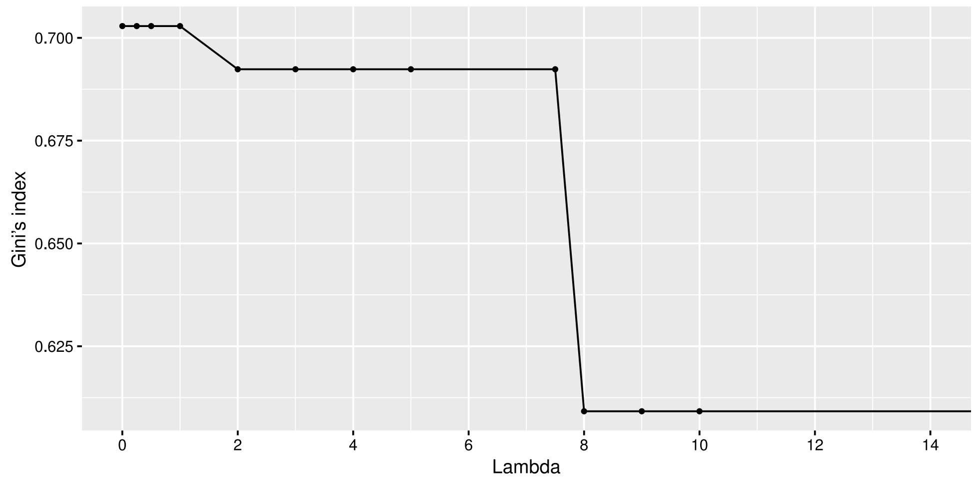

Fig. 4. The Gini index of the optimal solution, as a function of λ, at a higher resolution. For the Quarter Cost variation with a budget of 300.

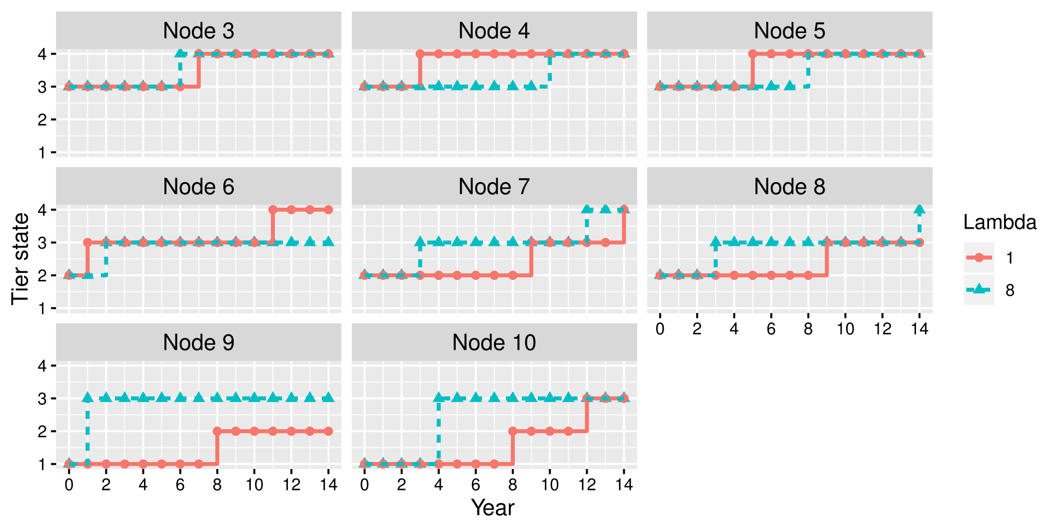

Fig. 5. Two solutions of the optimization problem with 10-nodes, for two distinct λ values {1,8} of the Quarter cost variation with budget 300.

5.1.2. *Comparison of the optimal, lns, and greedy solutions*

To compare the performance of LNS and CPLEX, we executed the LNS algorithm on 10-node instances, imposing a time limit of 15 min per instance. The results indicate that the LNS metaheuristic achieves a solution that is on average 18% higher than the optimal solution (obtained by CPLEX).

Fig. 6 illustrates the percentage decrease in the objective function for both the LNS metaheuristic and the CPLEX optimal solution, starting from the greedy solution (categories on the x-axis), along with the number of instances falling within each decrease range (on the y-axis). We note that in most cases, both LNS and CPLEX successfully reduced the greedy solution by 40%–60%. However, CPLEX exhibits an advantage in the 40%–60% category and also in the 60%–80% category, while LNS is more prevalent in the 0%–20% and 20%–40% categories.

This comparison between LNS and CPLEX cannot be performed on larger grids since the CPLEX cannot solve large problems. The following section extends our analysis to another numerical study focusing on the LNS solution method for larger grids.

5.2. *Power grid expansion in Myanmar*

Myanmar is a developing country that lies in Southeast Asia. Myanmar's population is roughly 54 million. As of 2019, it was estimated that only around 60% of the country's population had access to electricity; see the [23]. Myanmar's authorities, along with the World Bank, have initiated a "National Electrification Project" aimed at increasing the population's access to electricity by means of grid connectivity and off-grid solutions such as PV cells, see the [8]. The plan includes cumulative investments of up to $6 billion by 2030. The project aims to bring electricity to everyone in Myanmar by 2030.

Using public data available on [7] (2014–2015) for Myanmar's population and existing medium voltage power grid infrastructure at that time, we analyzed the expansion plan for Myanmar that would minimize the total weighted deprivation-equity cost, according to our model. The data we are using contains:

- The coordinates of medium voltage transmission lines throughout Myanmar.
- A list of all Myanmar cities and villages, including location (coordinates) and population size.

Unfortunately, the mentioned data sources do not include crucial information such as:

- Current tier states, i.e., which settlements have electricity, and in what form (on-grid or off-grid).
- Upgrade prices for moving from one specific tier to another.
- Deprivation costs for the population.

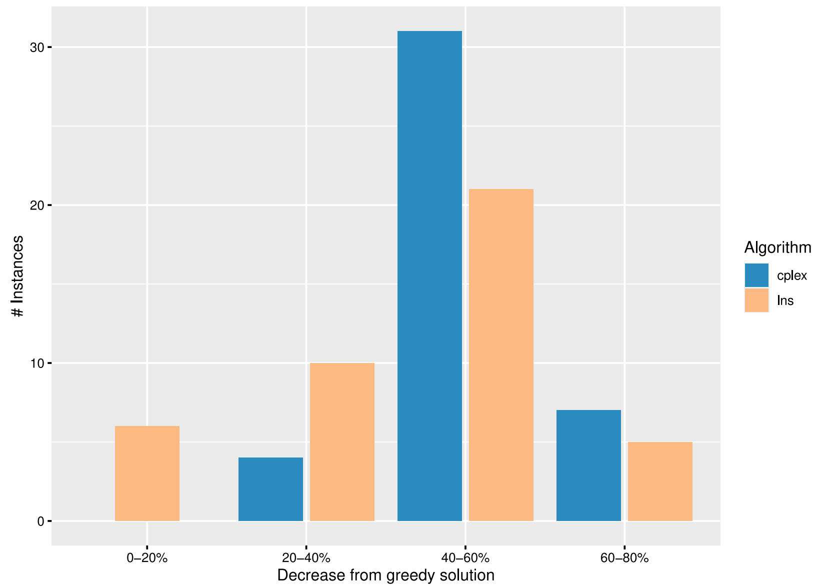

Fig. 6. A comparison of the CPLEX and LNS solution methods on 10-node instances.

In place of the missing data, we made several assumptions. First, we ran the KMeans clustering algorithm on the network to scale down the number of decision variables to a tractable size: from about 50k nodes representing cities, villages, and other types of settlements to either 100 or 200-clustered nodes (i.e., 100-node instance and 200-node instance respectively). Clusters were then assigned an initial tier according to their distance from the power grid's transmission lines. The population size of each cluster was determined according to the actual population at the nodes that are associated with the cluster. Upgrade costs of the clusters were set as a function of the cluster's distance from the power grid's transmission lines and of the population size that the cluster represents.

- The planning horizon was set as 15 years.
- We examined two sets of deprivation cost parameters for the 100-node instance ("base deprivation cost" using $\theta \in \{0.05, 0.04, 0.03, 0\}$, and these θ values multiplied by two, i.e. "double the deprivation cost"). For the 200-node instance, we used the base deprivation cost set.
- We used three budget levels for each instance.
  - For the 100-node instances, the annual budget levels were set as 401, 1053, and 1705 (per year). The high budget level is equivalent to the budget required to upgrade all nodes to the highest tier (tier four), if it is provided as a lump sum (aggregated instead of annually), the lowest budget level is equivalent to upgrading all nodes to tier three (or higher), and the middle budget level is their average.
  - For the 200-node instances, the annual budget levels were set as 441, 883, and 3494 (per year). The high budget level is equivalent to the budget required to upgrade all nodes to the highest tier (tier four), the medium budget level is equivalent to upgrading all nodes to tier three, and the lowest budget level was set as half the budget of the medium budget level.
- Five values of $\lambda \in \{0, 0.5, 1, 2.5, 4\}$.

Additional technical details, data pre-processing procedures, and assumptions we have made to generate the problem of the Myanmar case study are provided in Appendix B.

We now analyze the results of the LNS.

5.2.1. The Myanmar case study: Results

We ran each instance with a compute time limit of 10 h for 100-nodes and 20 h for 200-nodes. To analyze the LNS-heuristic's ability to identify good solutions, we compare the objective value of the LNS result to the value obtained by the first step of the algorithm, which corresponds to the greedy solution. We note that the greedy algorithm is not meant as a benchmark against the LNS solution quality but rather as an indication that the LNS algorithm can improve a naive greedy approach. Fig. 7 shows the improvement of the heuristic over the initial solution (in the y-axis) as a function of the equity parameter λ (the x-axis).

We observe a large improvement over the greedy heuristic, mostly for $\lambda = 0$ (between 17%–58% decrease, and about 40% on average). This improvement generally decreases when λ is increased. While it is unclear what causes this performance difference when λ changes, we believe this is possibly due to the characteristics of the greedy solution, which performs well when a higher weight is assigned to the equity portion of the objective function.

Another observation is that as the budget decreases, the improvement that the heuristic achieves is generally lower. This is probably because the heuristic's search space is limited at lower budget levels, and there are not that many improving solutions. When comparing the 100-node to the 200-node solution improvements, we observe that the latter improvement is smaller. This is because even though the search space is much larger for 200-nodes, the heuristic may reach a smaller portion of it (compared to 100-nodes). Finally, there is no significant difference in performance comparing the 100-node and the 100-node double deprivation cost instances.

Fig. 8 shows the convergence of the LNS for the 100-node instance and $\lambda = 0$. The x-axis shows the iteration number (in a logarithmic scale), and the y-axis shows the decrease from the initial (greedy) solution. We observe that even though the heuristic keeps improving the solution throughout its entire run-time, the rate of improvement decreases over time, i.e., the chart shows a somewhat linear trend in $\log(\text{iteration})$. This behavior is typical in additional instances we examined (except for cases where the LNS did not improve upon the greedy solution).

To analyze the development of tiers as a function of time, we have computed the average tier of nodes in two of the 100-node instances. The nodes are grouped into three types: those starting at Tier 1, Tier 2, and Tier 3. Fig. 9 includes the average tier of nodes of the three types: nodes that start at Tier 1 are on the upper row (two sub-figures),

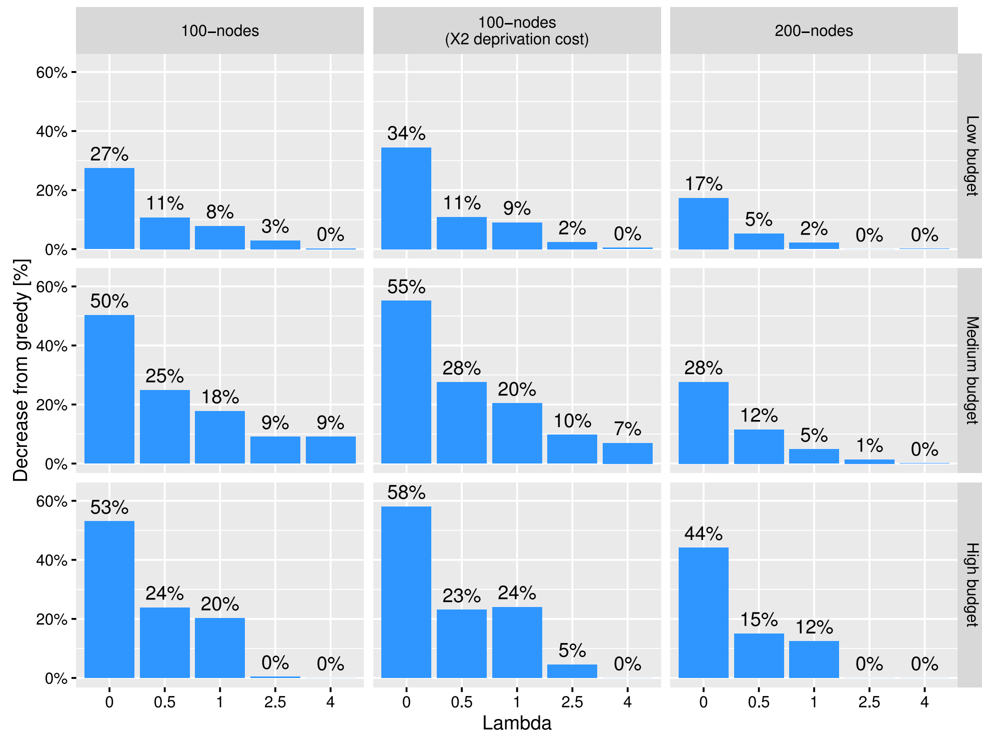

Fig. 7. The decrease in the objective value of the LNS-heuristic's final iteration versus the initial objective value (greedy solution).

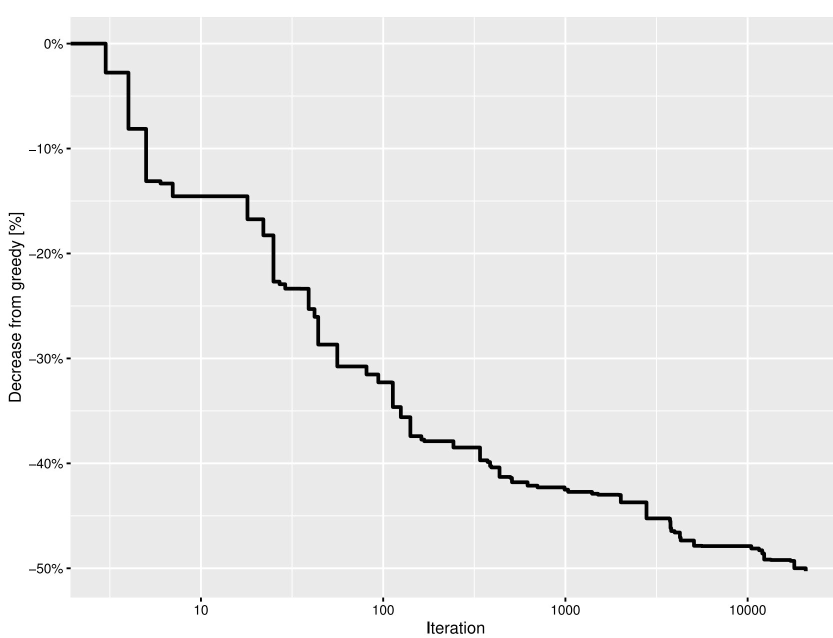

Fig. 8. The convergence of the 100-nodes instance, for $\lambda = 0$ and medium budget.

nodes that start on Tier 2 are in the second (middle) row, and nodes that start on Tier 3 are in the third (lower) row. The figure illustrates the development of nodes of each type as a function of the year, for $\lambda \in \{0, 4\}$ and two budget levels (low budget level on the left column and high budget level on the right column).

Several noteworthy observations can be made from the figure. First, as the parameter λ increases, the nodes that initially belong to lower tiers are upgraded earlier and to higher levels. This trend can be observed especially in the top row. In both budget scenarios, for $\lambda = 4$, the Tier 1 nodes undergo upgrades within the first few years, while for $\lambda = 0$, such upgrades are delayed. For Tier 2 and Tier 3 nodes, the two lines (for the different λ values) are closer to one another and are less affected by the λ value. These observations are similar to the observation about the small instance in Fig. 3.

Second, notable differences can be observed between the two budget levels. With a high budget, upgrades can reach all nodes' maximum or nearly maximum tier level. In contrast, lower budget levels limit upgrading capabilities, even by the end of the planning horizon.

Third, when comparing the different tiers on the left column for the case of $\lambda = 4$, it is noteworthy that the Tier 1 nodes (top row) end up being upgraded to an average tier slightly above 3, while the Tier 2 nodes (middle row) are upgraded to an average tier slightly above

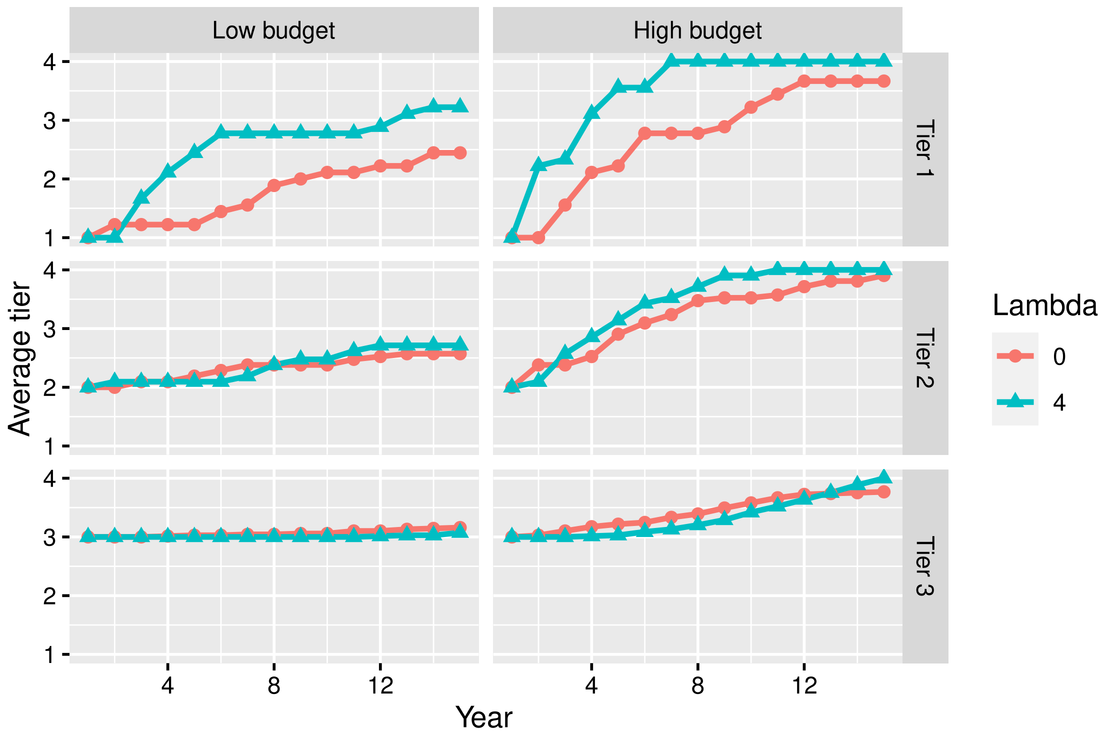

Fig. 9. Average tier in the 100-node instances.

2.5. This phenomenon is intriguing because the second row represents a greater population than the top row, and one would expect their average tier to be either higher or the same (as observed when $\lambda = 0$). However, due to the higher deprivation cost accumulated by the nodes in the top row during the initial years of the planning horizon (owing to their lower starting point), they are upgraded to a higher level to offset that deprivation cost (for equity). This highlights a memory property for the deprivation cost in our modeling approach due to the accumulation of the deprivation costs over time.

This prompts intriguing policy implications and points for discussion. One key consideration is the timeframe over which policymakers should account for the accumulated deprivation cost within the population: policymakers can either employ a long-term memory approach, taking into consideration historical deprivation costs, or a short-term memory approach. Another consideration is how far into the future policymakers project. Future models can explore this perspective by incorporating a discount factor on the yearly deprivation cost.

6. Conclusions

In this article, we studied the problem of power grid expansion planning from both effectiveness and equity perspectives. Our model minimizes an objective function which balances the deprivation costs that aim to reach effective solutions with the Gini mean absolute difference that aims toward fair (equitable) solutions. This balancing is performed by a weight parameter that multiplies the equity elements of the objective. This approach is especially useful in developing countries where some of the population has yet to be connected to the power grid. To the best of our knowledge, this is the first work considering both objectives in a power grid expansion optimization problem.

We first formulated the problem as a MIP and then solved it on a 10-node arbitrary instance. We varied some of the parameters, such as budget, deprivation cost, and equity weight, to gain insights into the solutions obtained and their influence on the Gini index. The results demonstrate that as the weight of equity increases (the multiplier λ), the optimal solution has a non-increasing equity measure (Gini index), as expected by Lemma 1.

We then developed an LNS search heuristic for solving large instances and tested it on instances with 100 and 200-nodes that were generated from real-world data based on the Myanmar power grid. The results demonstrate that the LNS can significantly improve over a naive, greedy approach. However, this is mainly true for lower values of λ.

Grid planners can use the model we developed to resolve various strategic dilemmas. For example, the model balances main grid connectivity and upgrades to densely populated areas relative to remote areas that use simple, local solutions such as photovoltaic cells or microgrids. Another strategic question is whether the planner should invest in small incremental steps that provide fast results or large ground-breaking steps (e.g., significant infrastructure enabling full grid connectivity) that provide long-term results. In general, investing in significant upgrades means that it takes longer to reach a desired grid state, i.e., one that is both effective and equitable. In contrast, investing in small incremental steps means a better solution is obtained in the short run, but reaching the desired grid state might not be achievable within the planning horizon. It is not an easy task to figure out how to balance the two, and the model suggested in this work can provide some guidance. Further developments to the model can include a bi-objective paradigm (an efficiency frontier), instead of a single objective with a weight parameter.

The models and solution methods presented in this work can be applied to a wide range of use cases of infrastructure planning, in which the planning problem consists of populations that obtain different tier levels, either within cities or nationwide. Some examples are internet infrastructure, road maintenance, cellular infrastructure, healthcare, and more.

Declaration of competing interest

None.

Data availability

Data will be made available on request.

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All authors approved the version of the manuscript to be published.

Appendix A. Piecewise linearization of the $g_{it}(D_{it})$ functions

The formulation we presented in Section 3 is a non-linear integer programming formulation due to the objective function, which introduces a convex term for the deprivation cost and the absolute value within the Gini mean absolute difference. We now show how to linearize the two expressions in the objective function.

First, we discretize the deprivation cost function to represent the deprivation cost at the beginning of each year. Since, in the original formulation, we allow for node upgrades only at the beginning of each year, this linearization of the objective coincides with the original objective's value (in the original formulation).

Second, we also linearize the absolute value terms in the objective function (inside the Gini mean absolute difference). The standard linearization method of absolute value we use here makes the optimal solution of this linearized version coincide with the optimal solution of the original problem.

Define the decision variables v_{itm} as:

$$v_{itm} = \begin{cases} 1 & \text{If node } i \text{ has been in tier } t \text{ for at least } m \text{ years} \\ 0 & \text{Otherwise} \end{cases}$$

Define the decision variables μ_{itm} as:

$$\mu_{itm} = \begin{cases} 1 & \text{If node } i \text{ has been in tier } t \text{ for exactly } m \text{ years} \\ 0 & \text{Otherwise} \end{cases}$$

The decision variables Z_{ij} will hold the absolute value of the deprivation cost difference between node i and node j in the optimal solution.

To define v_{itm} we use the following constraints:

$$v_{itm} \geq \sum_{y=k}^{k+m-1} X_{ity} - m + 1 \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, m \in \mathcal{Y}, k \in \{0, \dots, Y-m\} \tag{A.1}$$

In other words, if there is a combination of (at least) m consecutive years in which node i is at tier t, then v_{itm} will equal one.

Set μ_{itm} as an indicator variable such that $\mu_{itm} = 1$ if and only if node i was at tier t for exactly m years. To define μ_{itm}, we use the following constraints:

$$\mu_{itm} = v_{itm} - v_{itm+1} \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, m < Y \tag{A.2}$$

$$\mu_{itY} = v_{itY} \qquad \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{A.3}$$

$$\sum_{m=0}^{Y} \mu_{itm} = 1 \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{A.4}$$

We can see the validity of Constraints (A.4) by examining the expression $v_{itm} - v_{itm+1}$. It will equal one if and only if node i was exactly m years in tier t:

$$v_{itm} - v_{itm+1} = 1 \iff$$
$$v_{itm} = 1 \text{ and } v_{itm+1} = 0 \iff$$
$$\exists k \text{ such that: } \sum_{y=k}^{k+m} X_{ity} = m \quad \text{and} \quad \forall k : \sum_{y=k}^{k+m+1} X_{ity} \leq m$$

However, this condition is insufficient since we have not linked the μ_{itm} variables to transitions between different tiers. In other words, for every i, we must make sure that the total number of years depicted in all combinations of μ_{itm} does not exceed the total planning horizon of the problem:

$$\sum_{t=1}^{T} \sum_{m=0}^{Y} m\mu_{itm} = Y, \forall i \in \mathcal{N} \tag{A.5}$$

And we define the range of the v_{itm} variables:

$$v_{itm} \in \{0,1\} \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, m \in \mathcal{Y}. \tag{A.6}$$

The updated objective function then becomes:

$$\min \sum_{t,i,m} \pi_i g_{it}(m) \mu_{itm} + \Delta_{\text{Gini}} \tag{A.7}$$

where the Gini mean absolute difference is given by:

$$\Delta_{\text{Gini}} = \sum_{i,j \in \mathcal{N}} \pi_i \pi_j \left| \sum_{t,m} g_{it}(m) \mu_{itm} - \sum_{t,m} g_{jt}(m) \mu_{jtm} \right| \tag{A.8}$$

To incorporate Δ_{Gini} into the objective function (while keeping our problem a mixed integer linear programming), we define a set of variables Z_{ij} with the following set of constraints:

$$Z_{ij} \geq \sum_{t,m} g_{it}(m) \mu_{itm} - \sum_{t,m} g_{jt}(m) \mu_{jtm} \qquad \forall i,j \in \mathcal{N} \tag{A.9}$$

$$Z_{ij} \geq -\left(\sum_{t,m} g_{it}(m) \mu_{itm} - \sum_{t,m} g_{jt}(m) \mu_{jtm} \right) \qquad \forall i,j \in \mathcal{N} \tag{A.10}$$

This formulation is a standard technique that is used when absolute values appear in a mixed integer linear setting. It allows us to retain the linearity: Z_{ij} are always non-zero, and they will attain the absolute value of the expression $\sum_{t,m} g_{it}(m)\mu_{itm} - \sum_{t,m} g_{jt}(m)\mu_{jtm}$. Since the objective is a minimization problem, they will not exceed the absolute value of this expression.

The final updated objective function of our problem is of the form:

$$\min \sum_{t,i,m} \pi_i g_{it}(m) \mu_{itm} + \lambda \sum_{i,j} \pi_i \pi_j Z_{ij} \tag{A.11}$$

subject to the original constraints (5)–(12) along with the new constraints (A.1)–(A.10).

Appendix B. The Myanmar case study — assumptions and pre-processing

This appendix describes how we processed the Myanmar data set and the assumptions we made to generate the Myanmar optimization instances.

First, we extracted transmission line coordinates and population coordinates from [7], specifically the two data sets titled: "Myanmar - Cities and Town Location with Population" and "Myanmar - Existing Grid Medium Voltage Line Data". Then, the various villages and cities were clustered together using a KMeans clustering approach, using the standard `kmeans` function [24]. The cluster centers will represent the cluster and will be the new nodes we will consider in our optimization. These nodes (for the 100-node instance) are depicted in Fig. B.10, subfigure (A).

Fig. B.10 (subfigure B) illustrates the power grid transmission line infrastructure. It can be seen that the medium voltage transmission lines do not extend to the edges of the map, and for the most part, medium voltage transmission lines are necessary to provide grid connectivity to consumers. Hence, by measuring the distance from an area to the nearest transmission line, we can assume it is either connected or disconnected from the power grid (as the distance from the nearest medium voltage transmission line increases, it is less likely the node is connected to the power grid).

The population size at each node is determined according to the total population in the original elements (cities and settlements) that comprise the respective cluster. The distance between a cluster and the medium voltage transmission infrastructure is defined as follows:

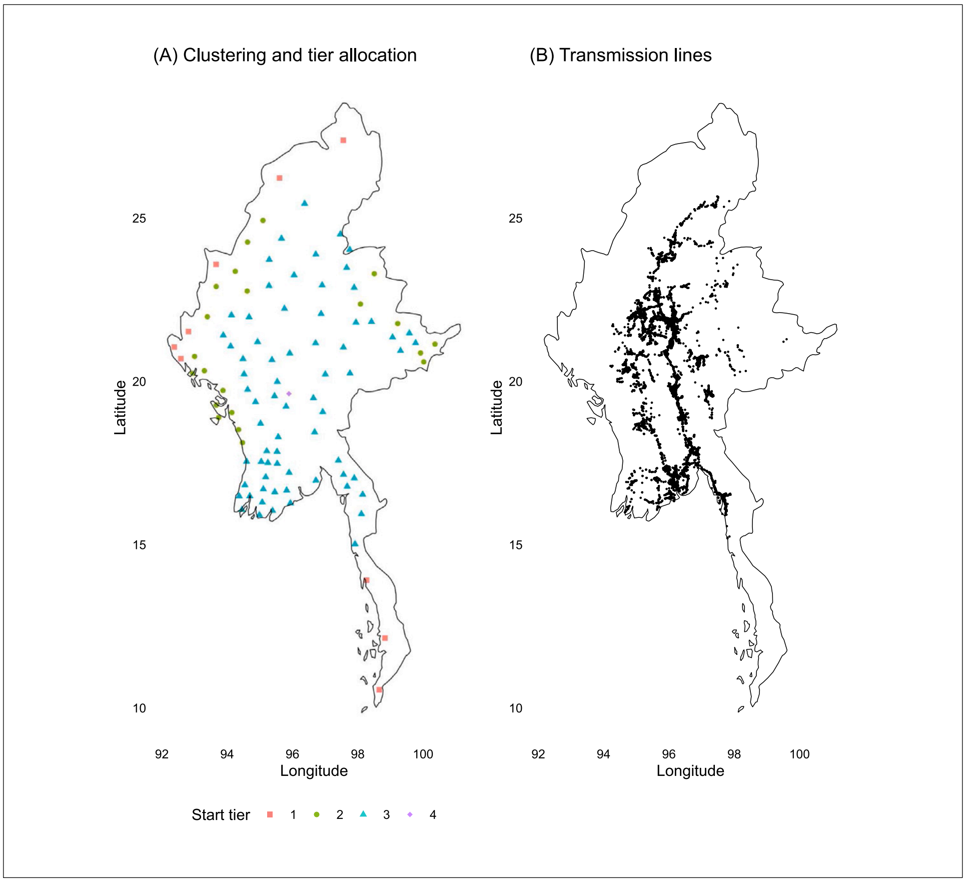

Fig. B.10. (A) The clustering and tier allocation results for the 100-node instances, represented on the Myanmar map. (B) The medium voltage transmission line infrastructure. Each point represents an element in the transmission line network.

the cluster is broken into its elements (original villages and cities). The shortest distance from each element to the medium voltage transmission line infrastructure is computed, and their average is set to be the distance. The tier is determined according to the above cluster's distance from the medium voltage transmission infrastructure. Nodes that are on the grid (up to 10 km from an existing medium voltage transmission line coordinate, i.e., the existing grid) were set as Tier 4 (full grid connectivity), and we controlled the algorithm so that these points will cluster into a single node (and the rest 99 nodes or 199 nodes are at a lower tier, in the case of the 100-node instance or 200-node instance, respectively). Nodes that are further than 150 km from the grid were set as Tier 1, nodes between 50–150 km from the grid were set as Tier 2, and nodes between 10–50 km from the grid were set as Tier 3.

For the upgrade cost model, we used either the node's distance from the grid if upgrading to tier four or the population size at the node if upgrading to a tier lower than four. This logic is derived from the fact that tier four is full grid connectivity and requires a transmission line setup from the node to the power grid. Usually, the setup of a transmission line is a function of the distance. The upgrade to lower tiers (one through three) does not require grid connectivity. Still, its scale is related to the population size (i.e., if installing household photo-voltaic cells or micro-grids).

The final upgrade prices were set according to Formula (B.1):

$$U_{iy}(s,t) = \begin{cases} \ln(c_{s,t}) \times (\text{population}) + k_{s,t} & \text{If } t < 4 \\ \ln(c_{s,t}) \times (\text{distance to grid}) + k_{s,t} & \text{If } t = 4 \end{cases} \tag{B.1}$$

The coefficients $c_{s,t}$ are presented in Table B.4.

Table B.4
Upgrade cost coefficients for the 100 and 200-node instances in Myanmar.

From tier s	To tier t	Coefficient $c_{s,t}$	Constant $k_{s,t}$
1	2	2.5	1
1	3	15	5
1	4	30	15
2	3	15	5
2	4	25	15
3	4	20	5

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